THE VELOCITY GRADIENT METHOD FOR THE MEASUREMENT OF VISCOUS PROPERTIES OF NON-NEWTONIAN LIQUIDS*

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ABSTRACT A method has been described which admits of a direct measurement of velocities at different depths in a flowing liquid. The liquid is contained in a rectangular cell of depth of the order of 1 mm. and the velocities measured with a microscope using an objective with a large working distance. Velocity distribution curves have been obtained with a sugar solution, two gelatin sols and a bentonite suspension. The advantages of the method are discussed.

A large number of liquids, especially concentrated colloids, do not obey Newton's linear relation between shearing stress and rate of shear. They are designated as non-Newtonian liquids. A systematic study of the viscous properties of such liquids has both theoretical and practical interest. The earlier work of Hess,[1] Hatschek,[2] Bingham,[3] Ostwald[4] and others has been considerably extended in recent years[5,6,7,8] and several accounts are available.[9] Newton's equation for viscous fluids is :

$$dv/dz = F/\eta \qquad \dots (1)$$

where dv/dz is the rate of shear (velocity gradient) in cm./sec./cm., F the shearing stress in dynes/sq. cm. and η the coefficient of viscosity in dynes.sec./sq. cm.

Ostwald[4] assumed that in colloids like gelatin sols a structure exists in the undisturbed sol which is progressively broken down as the rate of shear is increased so that the apparent fluidity (reciprocal of viscosity) of the system gradually rises.

Ostwald's relation is :

$$dv/dz = F^n/\eta' \qquad \dots (2)$$

where n is a constant greater than unity and has been called the "coefficient of structure" and η' is a constant whose dimensions are different from that of η the coefficient of viscosity.

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According to Bingham[8] a minimum shearing stress is necessary to start flow in " plastic " systems beyond which the rate of shear is proportional to the shearing stress. Bingham proposed for them an equation of the form :

$$dv/dz = (F - f)/\eta. \tag{3}$$

He called, f, the yield value[9] of the system.

Many examples of both ' structure-viscous ' and ' plastic ' systems have been found in the literature while some show intermediate behaviour.[9]

In viscometric measurements, especially by the capillary method, the rate of shear varies from layer to layer over wide limits. While this variation can be easily taken into account in Newtonian liquids where the coefficient η is constant and has a definite significance, it is not easy to do so in the case of non-Newtonian liquids where the values given by different viscometers of such quantities as the yield value, apparent viscosity or mobility depend on the method and also on the dimensions of the apparatus where the same method is used. It is obviously desirable to measure the rate of shear (velocity gradient) at different layers in the viscometer. Pichot[10] and Philippoff[11] have described methods which permit observations of the velocity gradient, but these methods do not give the velocity gradient in absolute units. Kroepelin[12] uses a pitot tube for this purpose but his method is applicable for rather high rates of shear whereas measurements at low rates of shear are of especial interest, *e.g.*, in distinguishing between equations (2) and (3). His method also involves the use of fairly wide tubes which may complicate the conditions of flow. A flat cell of the type used in the microscopic measurement of cataphoretic velocities of colloids is a convenient form of apparatus which has been used by us at low rates of shear.

The flat cell used had the following dimensions : length—8.0 cm., breadth—1.0 cm., height—0.0644 cm.

For Newtonian liquids in a rectangular flat cell,

$$dv/dz = -\pi z/\eta \tag{4}$$

where π is the pressure gradient (negative in this case) and z the distance from the central plane.

From equations (2) and (3) we have for non-Newtonian liquids respectively :

$$dv/dz = -\pi^n z^n/\eta' \tag{5}$$

and

$$dv/dz = -\pi(z - z_0)/\eta \tag{6}$$

z_0 is the distance along the normal to the direction of flow measured from the central plane, up to which the fluid flows as a solid plug,[9] $-\pi z_0$ gives a measure of, and is equal to, the lower yield value.[9]

Measurements have been made on the following systems :

(*a*) a saturated sugar solution (67%),

(*b*) two gelatin sols, and

(*c*) a hydrogen bentonite suspension.†

Finely powdered pure sugar charcoal was suspended and the speeds of the particles at different depths were measured.

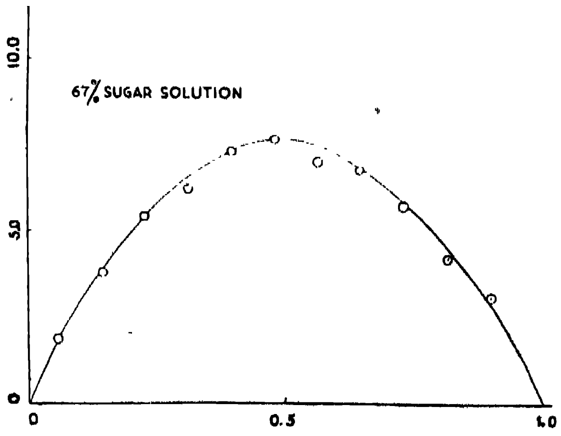

FIG. 1

(*a*) As is to be expected, the sugar solution (Fig. 1) gives a perfect parabolic distribution of velocities which is characteristic of Newtonian liquids. The pressure gradient was, however, very small and in the present form of the

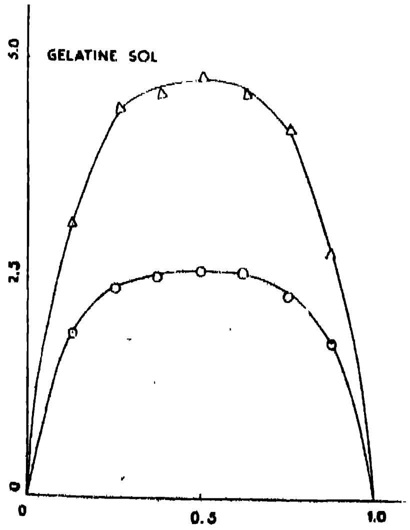

FIG. 2

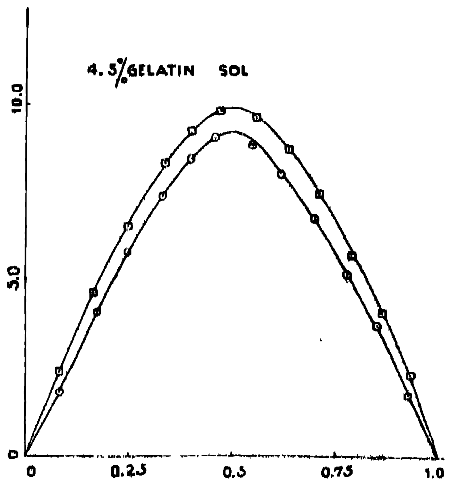

FIG. 3

† A very fine suspension of a bentonite, obtained by means of a Sharples supercentrifuge, was electrodialysed to give the hydrogen bentonite. The methods of preparation and other experimental details will be described elsewhere.

apparatus could not be measured very accurately. The coefficient of viscosity obtained from the approximate pressure gradient is about 2 poise. The viscosity of the same solution measured by means of a rotary viscometer is 1.7 poise which agrees with the value given in International Critical Tables.[13]

(*b*) The velocity distribution in concentrated gelatin sols gives curves of a higher degree than the parabola. The value of '*n*' in equation (5) calculated from the curves in Fig. 2 is about 2.8. Curves given in Fig. 3 are however parabolic in nature, and the behaviour of gelatin sols depends on concentration and other factors.

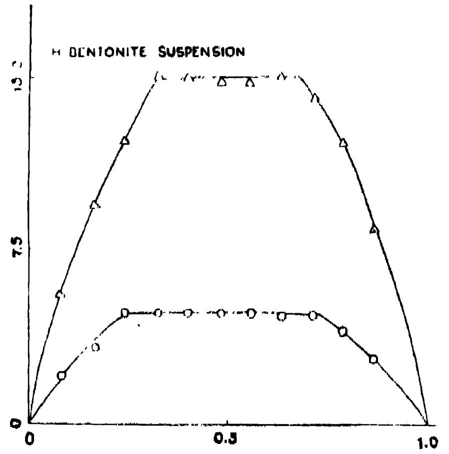

FIG. 4

(*c*) The bentonite suspension (Fig. 4, curves i and ii) shows a parabolic distribution of velocities with a central flat region whose depth diminishes with increase in the pressure gradient. z_0 is evidently equal to half the depth of the flat region. The yield values calculated from the two curves are 27 and 30 dynes/sq.cm. respectively. The yield value of the same suspension obtained by means of a capillary viscometer is 88 dynes/sq.cm. which is more than three times the value obtained from Fig. 4 while a rotary viscometer gives values nearly equal to those obtained by the flat cell.

It is evident that the method described here gives curves which are significantly characteristic of the variation of the velocity gradient with shearing stress. These curves are capable of yielding on mathematical analysis very useful information not obtainable by the usual viscometric methods. The apparatus is being modified for more accurate measurements at low rates of shear and at constant temperature.

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